

The **Pointing Error Analysis Tool** is a set of Matlab functions that are used to evaluate the Standard Pointing Error Metrics from time-domain and frequency-domain data. The pointing error metrics are called the **accuracy metric, displacement metric, jitter metric, stability metric,** and the **windowed stability metric**

. These metrics are based on unambiguous mathematical models involving the point spread function of optical instruments, and therefore relate the pointing error directly to instrument performance. The mean-square pointing error is expressed by exactly equivalent equations in the time domain and in the frequency domain.

The Pointing Error Analysis Tool is a set of Matlab functions that are used to evaluate the Standard Pointing Error Metrics from time-domain and frequency-domain data. Other functions aid in evaluating the contribution of sensor noise on the pointing error metrics.

The Pointing Error Analysis Tool has the following features and benefits

- Quickly analyze time-domain, frequency-domain, and stochastic (random) pointing error.
- Determine the relative contribution of various disturbance sources.
- Determine which are the most offending disturbances in a pointing system.
- Evaluate the pointing error metrics for a variety of window times without having to recompute everything.

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## Pointing Error Definitions

**Pointing Error** is the angular rotation from the desired pointing direction to the actual pointing direction, or somewhat more loosely it is the difference between the actual pointing direction and the desired pointing direction.

**Accuracy** is the root-mean-square (RMS) pointing error of the line of sight (LOS) over any interval of time (window width  $T$   $a \rightarrow \infty$ ). Rotation about the LOS may also be considered. The three body axes may be considered rather than the LOS. The RMS accuracy is designated ( $\sigma$   $a^2 + \mu^2)^{1/2}$ .

**Displacement** (or Shift) is the average pointing error of the LOS within an interval of  $T_d$  seconds. The Displacement Metric is used to compare the average pointing direction of one image with another.

**Jitter** is the RMS pointing error of the LOS within an interval of  $T_j$  seconds. Jitter is usually specified to limit blur or streaking of an image during the exposure time, to limit signal variation in a sensor, or to limit some other mensuration (measurement) error.

**Stability** is the RMS change in the LOS from one end of an interval of  $T_s$  seconds to the other end. Stability is usually specified for image registration or for data correlation from one time to another.

**Windowed Stability** is the RMS change in displacement from the center-of-integration time of one measurement to the center-of-integration time of another measurement, where the displacement is the average attitude within a window of width  $T_d$ . The time between the center-of-integration times is designated  $T_s$ .

The windowed stability metric therefore depends on both  $T_d$

and  $T_s$ .

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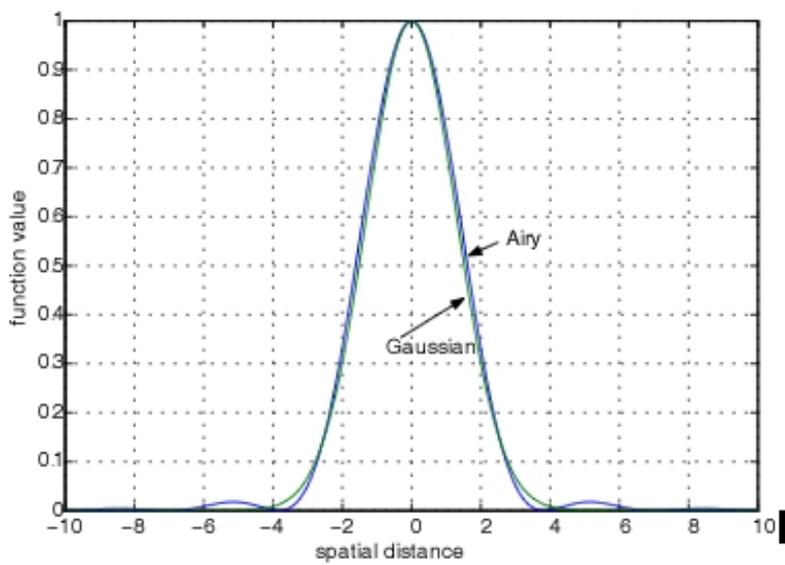
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## Concepts from Optics

The optical quality of an imaging system is limited partly by the quality of its optics. The point spread function (PSF) describes how a column of light is spread out after passing through the optics. The PSF is shown in the first figure below. The bright central region is called the Airy Disc. A cross section of the Airy Disc is shown in the second figure. This function can be approximated as a Gaussian  $G(x, \sigma_{psf}^2)$ . The effect of jitter motion on image quality can be determined by how it affects the PSF. Line-of-sight jitter motion can generally be modeled by as a Gaussian random error with variance  $\sigma_j^2$ . The effective PSF is given by the convolution of the Gaussian PSF and the Gaussian jitter, which is simply  $G(x, \sigma_{psf}^2 + \sigma_j^2)$ .

In effect, the jitter motion widens the PSF. The jitter affects the image only during an exposure

time of T seconds, so the jitter variance is computed over a window of T seconds.



## Point Spread Function with Airy Disc

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### Standard Pointing Error Metrics

The pointing error metrics are called the **Accuracy Metric**, **Displacement Metric**, **Jitter Metric**, **Stability Metric**, and the **Windowed Stability Metric**. Time domain expressions for the pointing error metrics can be written directly from the

corresponding definitions. The time-domain equations for the pointing error metrics are not used as a computational tool because they are computationally burdensome, and because the time-domain data does not usually fit a time window exactly. The pointing error metrics in the time domain are converted to the frequency domain by using Parseval's equality. The time-domain and frequency-domain expressions are, mathematically, exactly equivalent.

In the frequency domain, the pointing error metrics are given by the following expressions,

**Accuracy Metric**  $\sigma_a^2 = \int S(\omega) d\omega$

**Displacement Metric**  $\sigma_j^2 = \int S(\omega) W_j(\omega T_d) d\omega$

**Jitter Metric**  $\sigma_j^2 = \int S(\omega) W_j(\omega T_j) d\omega$

**Stability Metric**  $\sigma_s^2 = \int S(\omega) W_s(\omega T_s) d\omega$

**Windowed Stability Metric**  $\sigma_{sw}^2 = \int S(\omega) W_{sw}(\omega T_d, \omega T_s) d\omega$

where  $S(\omega)$  is the power spectral density of the attitude error, and  $T_d$ ,  $T_j$ , and  $T_s$  are the window times introduced in the pointing error definitions. The weighting functions  $W$

$W_d$

$W_j$

$W_s$ , and  $W_{sw}$

are given by

**Displacement Weighting Function**  $W_d(v) = 2(1 - \cos v)/v^2$

**Jitter Weighting Function**  $W_j(v) = 1 - 2(1 - \cos v)/v^2$

**Stability Weighting Function**  $W_s(v) = 2(1 - \cos v)$

**Windowed Stability Weighting Function**  $W_{sw}(v_1, v_2) = W_s(v_1)W_d(v_2)$

The expressions for the pointing error metrics may appear to be difficult to evaluate, but a very simple computational method is described in the next section. In the case of a few sinusoidal disturbances, these metrics are particularly easy to evaluate. An example for the case of reaction wheel imbalance is given in the references (see the Further Reading page of this article).

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### Computational Techniques for Evaluating the Pointing Error Metrics

The pointing error metrics are evaluated quickly and accurately from time-domain data by using fast computational algorithms that utilize the Discrete Fourier Transform to convert the time-domain data to the frequency domain. The frequency domain equations for the pointing error metrics are then evaluated by summing a weighted mean square response. A different weighting is used for each metric. Individual contributions to the pointing error metrics can be determined to determine the most offending disturbance sources and to determine sensitivities to disturbances.

Frequency-domain data can also be obtained from the frequency response of a transfer function. It is particularly fast and easy to evaluate the pointing error metrics for reaction wheel or CMG disturbances.

For stochastic (random) disturbances, one of two paths can be taken to evaluate the pointing error metrics. One path is to simulate random inputs in a time-domain simulation, and then evaluate the metrics using the time-domain response. The other path is to augment the frequency-domain plant model with weighting functions. These weighting functions are linear transfer functions that approximate the weighting functions in the pointing error metrics. The

covariance (or variance) at the output of the transfer functions is then evaluated by using standard methods available in Matlab. This path is often the fastest.

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### **Why Not Use the Peak Stability or Other Metric?**

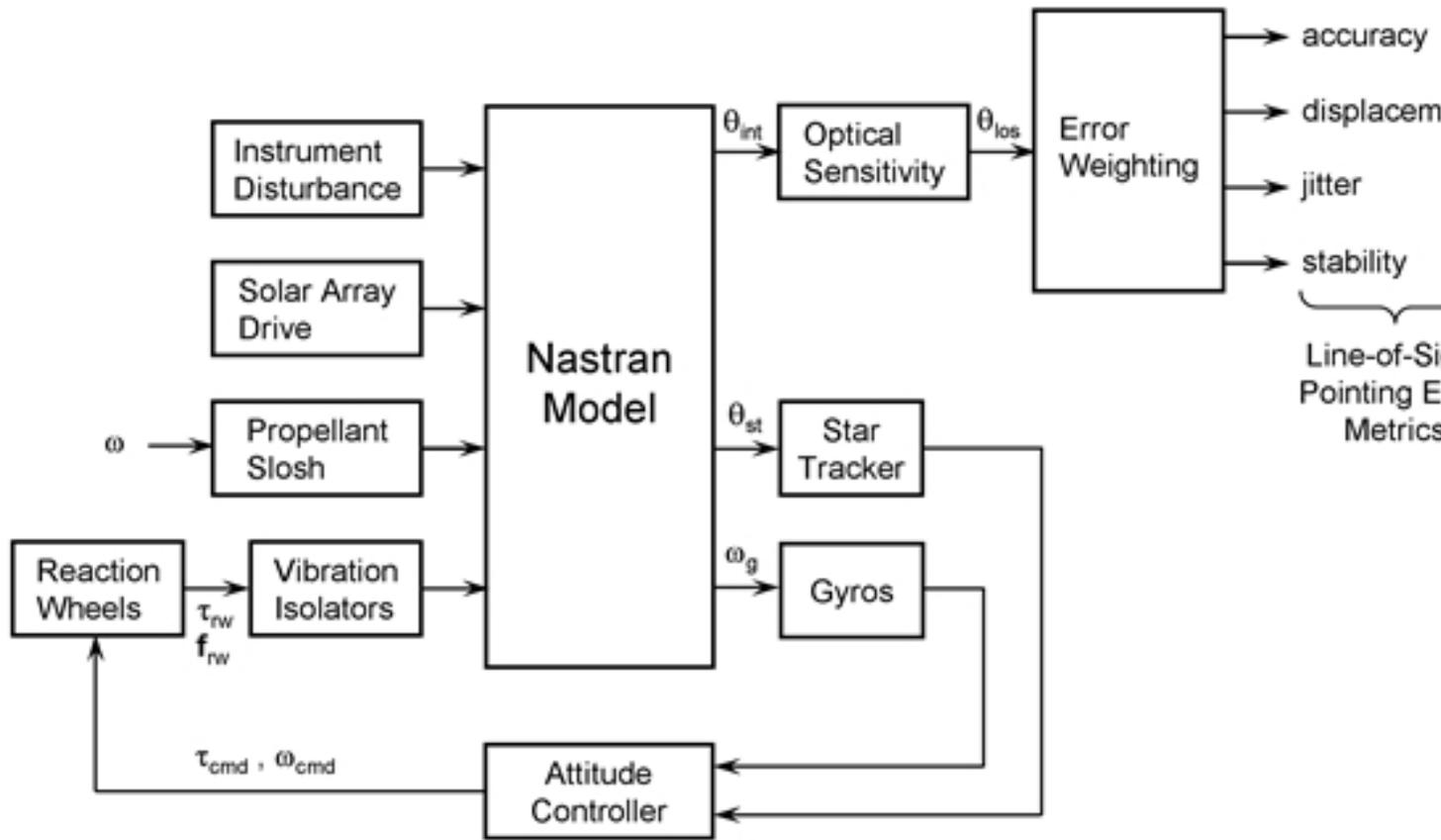
The peak stability metric and other metrics are very conservative, inconsistent, and they do not have any statistical meaning in connection with performance of an optical instrument. They also have no equivalence in the time domain and in the frequency domain. Furthermore, evaluation of the the peak stability metric and other metrics is computationally burdensome. See the discussion ["What is the peak stability metric and why is it conservative?"](#) in the FAQ for a detailed discussion.

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### **Pointing Error Analysis**

The figure below illustrates a flexible-body system with various inputs and outputs, including disturbances, actuator inputs, sensor outputs, and a payload line of sight. Particular nodes in the Nastran model correspond to the locations of the actuators, sensors, and payload. There can be other nodes of interest that define additional inputs and outputs, such as the locations of thrusters, accelerometers, the tip of a boom, or the end of a solar array.

The reduced-order Nastran model shown in the figure is obtained from the Structural Model Reduction tool. The closed-loop response is evaluated by using the CSI Analysis tool, and the pointing errors are evaluated by the Pointing Error Analysis tool.



Pointing Error Analysis

### Further Reading

The development and application of the Standard Pointing Error Metrics can be found in the following documents and references therein.

The Johns Hopkins University Applied Physics Laboratory, STEREO Guidance and Control System Specification, NASA Contract NAS5-97271, FSCM No. 88898, Drawing No. 7381-9310, February 2003.

" [Pointing Error Definitions, Metrics, and Algorithms](#) ", Paper No. AAS 03-559, AAS/AIAA Astrodynamics Specialists Conference, Big Sky, MT, 3–7 August 2003. In *Advances in the Astronautical Sciences*, Vol. 116, Part II, pp. 901–920, 2003. [CD, Book]

See the [Errata](#) sheet.

" [Definitions, Metrics, and Algorithms for Displacement, Jitter, and Stability](#) ", Flight Mechanics Symposium, NASA Goddard Space Flight Center, NASA/CP-2003-212246, 28–30 October 2003. [CD]

See the [Errata](#) sheet.